Spiral cylindrique sans courbes terminales

Perturbations causées par l'inertie du spiral

Caractéristiques du spiral

Référence :E:\Résonateur (TA)\Data\Bal_spiral cylindrique (ex num).mcd(R)

Dimensions $\quad ép = 0.09\,mm \quad ha = 0.334\,mm \quad S = 0.03\,mm^2 \quad R_0 = 5\,mm$

Elinvar $\quad \rho_s = 8\times 10^3\ kg\cdot m^{-3} \quad E = 1.7\times 10^{11}\,Pa \quad G = 6.538\times 10^{10}\,Pa$

Partie cylindrique $\quad n_s := 10.15 \quad \psi_0 := n_s\cdot 360\cdot deg \quad \psi_0 = 3.654\times 10^3\,deg \quad L := R_0\cdot\psi_0 \quad L = 318.872\,mm$

Forme du spiral en fonction de l'élongation du balancier $\quad \psi(\theta) := \psi_0 + \theta \quad R(\theta) := \dfrac{L}{\psi(\theta)}$

$x(\alpha,\theta) := R(\theta)\cdot cos(\alpha) \quad y(\alpha,\theta) := R(\theta)\cdot sin(\alpha) \quad s(\alpha,\theta) := R(\theta)\cdot\alpha \quad \alpha_0 := 120\cdot deg$ (valeur de test)

Calcul de la variation du centre de masse

$$\xi(\alpha,\theta) := \frac{R(\theta)^2}{s(\alpha,\theta)}\cdot\int_0^{\alpha} cos(\alpha')\,d\alpha' \qquad \eta(\alpha,\theta) := \frac{R(\theta)^2}{s(\alpha,\theta)}\cdot\int_0^{\alpha} sin(\alpha')\,d\alpha' \qquad \xi(\alpha_0,\theta_0) = 1.925\,mm$$

$$\eta(\alpha_0,\theta_0) = 3.335\,mm$$

$$\alpha_G(\alpha) := \frac{\alpha}{2} \qquad OG(\alpha,\theta) := 2\cdot\frac{R(\theta)}{\alpha}\cdot sin\left(\frac{\alpha}{2}\right) \qquad \xi(\alpha,\theta) := OG(\alpha,\theta)\cdot cos(\alpha_G(\alpha)) \qquad \xi(\alpha_0,\theta_0) = 1.925\,mm$$

$$\eta(\alpha,\theta) := OG(\alpha,\theta)\cdot sin(\alpha_G(\alpha)) \qquad \eta(\alpha_0,\theta_0) = 3.335\,mm$$

Calcul de la variation du moment d'inertie

$$\rho(\alpha,\theta) := \sqrt{(x(\alpha,\theta)-\xi(\alpha,\theta))^2 + (y(\alpha,\theta)-\eta(\alpha,\theta))^2} \qquad \rho(\alpha_0,\theta_0) = 4.31\,mm$$

$$\rho(\alpha,\theta) := R(\theta)\cdot\sqrt{1+\frac{4}{\alpha^2}\cdot sin\left(\frac{\alpha}{2}\right)^2 - \frac{4}{\alpha}\cdot sin\left(\frac{\alpha}{2}\right)\cdot cos\left(\frac{\alpha}{2}\right)} \qquad \rho(\alpha_0,\theta_0) = 4.31\,mm$$

$$J_s(\theta) := \frac{m_s}{L^3}\cdot\int_0^{\psi(\theta)} \rho(\alpha,\theta)^2\cdot s(\alpha,\theta)^2\cdot R(\theta)\,d\alpha \qquad J_s(0) = 6.476\,mg\cdot cm^2 \qquad J_s(\theta_0) = 5.616\,mg\cdot cm^2$$

$$J_s(\theta) := \frac{m_s\cdot L^2}{\psi(\theta)^5}\cdot\left(\frac{\psi(\theta)^3}{3} + 2\cdot\psi(\theta) + 2\cdot\psi(\theta)\cdot cos(\psi(\theta)) - 4\cdot sin(\psi(\theta))\right) \qquad J_s(\theta_0) = 5.616\,mg\cdot cm^2$$

Par développement en série $\quad n := 0, 2 .. 6$

$$g(\alpha,n) := \int_0^{\alpha} (\alpha-\alpha')^n\cdot\alpha'\cdot cos(\alpha-\alpha')\,d\alpha' \qquad f(\theta,n) := \frac{2\cdot(-1)^{\frac{n}{2}}}{(n)!\cdot\psi(\theta)^{n+5}}\cdot\int_0^{\psi(\theta)} (\psi(\theta)-\alpha)\cdot\alpha\cdot g(\alpha,n)\,d\alpha$$

$$\mathbf{A}(\theta) := (f(\theta,0)\ \ 0\ \ f(\theta,2)\ \ 0\ \ f(\theta,4)\ \ 0\ \ f(\theta,6))^T \qquad Jd_s(\theta) := m_s\cdot L^2\cdot\sum_n \left(\mathbf{A}(\theta)_n\cdot\theta^n\right)$$

$$\mathbf{A}(\theta_0)^T = \left(7.123\times 10^{-5}\ \ 0\ \ 5.292\times 10^{-9}\ \ 0\ \ 3.556\times 10^{-9}\ \ 0\ \ -1.218\times 10^{-10}\right)$$

$$Jd_s(0) = 6.476\,mg\cdot cm^2 \qquad Jd_s(\theta_0) = 5.658\,mg\cdot cm^2$$

Approximativement

$$A_0(\theta) := \frac{1}{3\cdot\psi(\theta)^2} \qquad A_0(\theta_0) = 7.107\times10^{-5}$$

$$A_2(\theta) := \frac{1}{\psi(\theta)^4}\cdot\left[1-\cos(\psi(\theta))+10\cdot\frac{\sin(\psi(\theta))}{\psi(\theta)}+20\cdot\left(\frac{1+2\cdot\cos(\psi(\theta))}{\psi(\theta)^2}\right)\right] \qquad A_2(\theta_0) = 5.287\times10^{-9}$$

$$A(\theta,n) := \frac{2\cdot(-1)^{\frac{n}{2}}}{(n)!\cdot\psi(\theta)^4}\cdot\left[\cos(\psi(\theta))-2\cdot(2\cdot n+1)\cdot\frac{\sin(\psi(\theta))}{\psi(\theta)}\right]\cdot(n>2) \qquad A(\theta_0,4) = 3.65\times10^{-9}$$

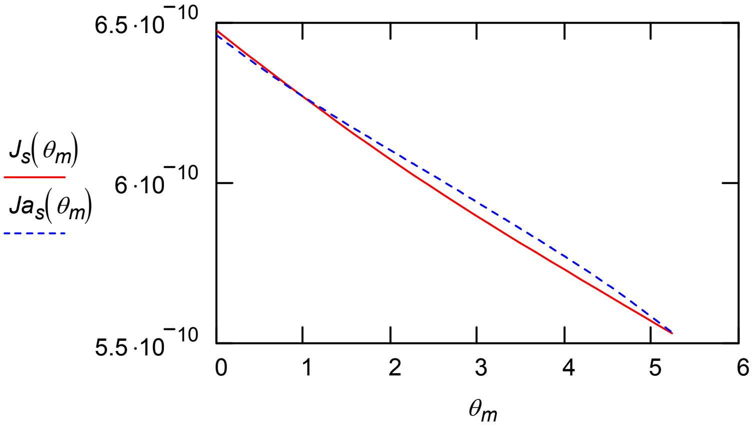
$$A(\theta_0,6) = -1.303\times10^{-10}$$

$$\mathbf{A}(\theta) := \begin{pmatrix}A_0(\theta) & 0 & A_2(\theta) & 0 & A(\theta,4) & 0 & A(\theta,6)\end{pmatrix}^T$$

$$\mathbf{A}(\theta_0)^T = \begin{pmatrix}7.107\times10^{-5} & 0 & 5.287\times10^{-9} & 0 & 3.65\times10^{-9} & 0 & -1.303\times10^{-10}\end{pmatrix}$$

$$Ja_s(\theta) := m_s\cdot L^2\cdot\sum_n\left(\mathbf{A}(\theta)_n\cdot\theta^n\right) \qquad Ja_s(0) = 6.461\,mg\cdot cm^2 \qquad Ja_s(\theta_0) = 5.641\,mg\cdot cm^2$$

$$\theta_m := 0\cdot deg, 10\cdot deg\,..\,300\cdot deg$$



Perturbation de marche

Moment d'inertie du balancier $J_b = 434.68\,mg\cdot cm^2$

Calcul de la perturbation de marche par intégration numérique

$$\delta_{Js}(\theta_0) := \frac{1}{2\cdot\pi\cdot J_b}\cdot\int_0^{\pi} J_s(\theta_0\cdot\cos(\varphi))\,d\varphi \qquad \delta_{Js}(\theta_0) = 7.5022\times10^{-3}$$

$$\mu_{Js}(\theta_0) := -86400\cdot\delta_{Js}(\theta_0) \qquad \mu_{Js}(\theta_0) = -648$$

Approximativement

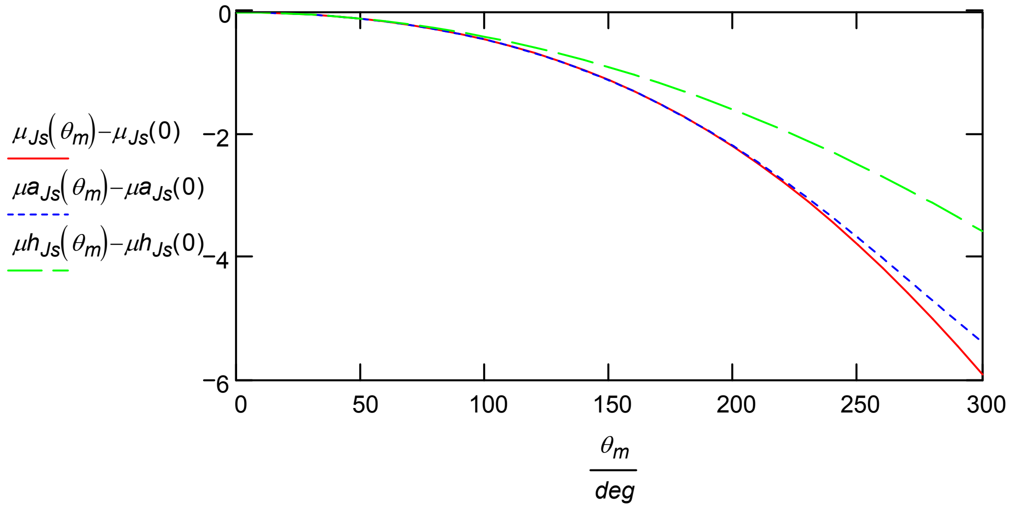
$$\delta a_{Js}(\theta_0) := \frac{m_s\cdot L^2}{2\cdot J_b}\cdot\sum_n\left[\frac{n!}{2^n\cdot\left(\left(\frac{n}{2}\right)!\right)^2}\cdot\mathbf{A}(0)_n\cdot\theta_0^{\,n}\right] \qquad \delta a_{Js}(\theta_0) = 7.4825\times10^{-3}$$

$$\mu a_{Js}(\theta_0) := -86400\cdot\delta a_{Js}(\theta_0) \qquad \mu a_{Js}(\theta_0) = -646$$

Selon Haag:

$$\delta h_{Js}(\theta_0) := \frac{m_s\cdot L^2}{6\cdot J_b\cdot\psi_0^{\,2}}\cdot\left[1+\frac{3}{2}\cdot\frac{\theta_0^{\,2}}{\psi_0^{\,2}}\cdot\left[1-\cos(\psi_0)+10\cdot\frac{\sin(\psi_0)}{\psi_0}+20\cdot\left(\frac{1+2\cdot\cos(\psi_0)}{\psi_0^{\,2}}\right)\right]\right]$$

$$\delta h_{Js}(\theta_0) = 7.4656\times10^{-3} \qquad \mu h_{Js}(\theta_0) := -86400\cdot\delta h_{Js}(\theta_0) \qquad \mu h_{Js}(\theta_0) = -645$$



Résolution de l'équation différentielle du mouvement

$I(\theta) := J_b + J_s(\theta)$ $\quad b(\theta) := \frac{1}{2 \cdot I(\theta)} \cdot \frac{d}{d\theta} J_s(\theta)$ $\quad c(\theta) := \frac{C}{I(\theta)}$ $\quad t_f := 4 \cdot T_0 \cdot \frac{1}{sec}$ $\quad t_f = 1.6$

$y := \begin{pmatrix} 2 \\ 50 \end{pmatrix}$ $\quad D(t, y) := \begin{bmatrix} y_1 \\ -b(y_0) \cdot (y_1)^2 - c(y_0) \cdot sec^2 \cdot y_0 \end{bmatrix}$ $\quad \omega_0 := \omega_0 \cdot sec$ $\quad D1(t, y) := \begin{pmatrix} y_1 \\ -\omega_0^{\ 2} \cdot y_0 \end{pmatrix}$

$\Theta := rkfixe(y, 0, t_f, 400, D)$ $\quad t := \Theta^{\langle 0 \rangle}$ $\quad \theta := \Theta^{\langle 1 \rangle}$ $\quad Z := rkfixe(y, 0, t_f, 400, D1)$ $\quad \theta 1 := Z^{\langle 1 \rangle}$